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SHORT-TERM INFLATION FORECASTING MODELS WITH BAYESIAN VAR: EVIDENCE FROM AZERBAIJAN

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SHORT-TERM INFLATION FORECASTING MODELS WITH BAYESIAN VAR: EVIDENCE FROM AZERBAIJAN[1]

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Abstract

In alignment with the Central Bank's objective to maintain price stability in the economy, this paper is dedicated to forecasting short-term inflation in Azerbaijan with various time series models including Bayesian Vector Autoregressive (BVAR) technique, which, alleviates overparameterization problem. We have also utilized autoregressive (AR) and standard VAR as benchmark models to make a comparison of forecast errors derived from out-of-sample analysis with those of BVAR model. For BVAR estimations, Litterman, Minnesota and Sims-Zha Normal Wishart (NW) prior have been employed. The monthly estimation period covers the date range from 2003M1 to 2019M6 and by using the expanding window strategy, we extended the data window to 24 months and forecasted the subsequent 24 months. We have carried out analysis in two stages: economic category-specific and incremental modelling. In a category-specific analysis, we developed 5 models for VAR and BVAR priors, each focusing on various sectors of the economy. We then applied an incremental approach, where variables from the earlier category-specific models were added step by step, enabling us to evaluate how the forecast performance changed as additional variables were included in the models. The study analyzed different frameworks for the assessment of the final forecast with the objective of improving the forecast accuracy. Overall, the examination of different models and estimation techniques demonstrates that each model and estimation technique have a significant contribution to our suggested four different strategies of forecast assessment.

Keywords: short-term inflation forecasting, AR, VAR and BVAR models

JEL: C51, C52, C53, E31, E37, F17

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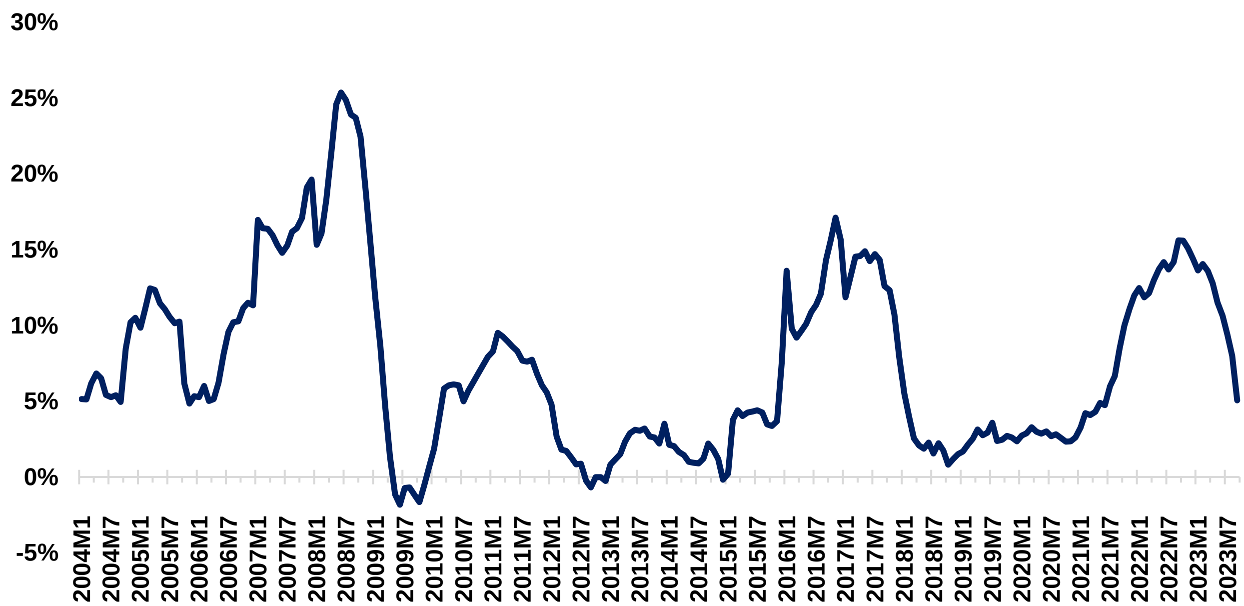
1. Introduction

Inflation-targeting monetary framework employed by central banks have been placing great emphasis on inflation forecasting. Accurate projections of inflation trends for price stability and financial stability are critical because monetary policy decisions affect the entire economy and financial sector. As a result, the past decade has witnessed the adoption of high-end econometrics estimation techniques such as BVAR models by central banks to improve forecasting accuracy. Traditional multivariate time series estimation methods have certain restrictions on employing over five variable frameworks that may yield biased inference due to over-parametrization and dimensionality problems. Bayesian VAR is a powerful technique to analyze and forecast dynamic relationships among macroeconomic variables. BVAR allows the integration of prior beliefs on the parameters and posterior information. This prior information can be based on historical data, economic theory—such as the Taylor Rule or the Phillips Curve—and expert opinion, all of which guide the model's assumptions and can lead to different outcomes. However, this study does not focus on the assessment of prior information from different sources. Prior beliefs can be utilized to limit the use of the less important variables and elevate the strength of relevant variables. Thus, Bayesian VAR offers a solution to potential dimensionality and over-parametrization problems.

The Central Bank of Azerbaijan Republic (CBAR) mainly targets 12-month inflation and forecasts for the next two years. That is why we have analyzed aggregate annual inflation employing different models. Various patterns of inflation dynamics in Azerbaijan were observed throughout this period. From 2004 to 2007, the headline inflation rate escalated to a double-digit and average annual inflation hiked to 10.3 per cent. The reason is explained by the demand pressures, driven mostly by the increase in fiscal expenditure due to a surge in oil windfall, high food inflation and a spike in administrative prices. Between 2009 and 2015, the economy enjoyed a stable period with a low inflation rate arising from stability in the international market. However, a substantial drop in oil prices at the end of 2014 caused a deficit in the current account, which in turn entailed the devaluation of the national currency in 2015. Reduced oil revenues have had a lagged effect on the economy. If average inflation was 3.3% between the 2009-2014 boom period, from 2015 to 2017, after devaluation episodes of local currency, inflation averaged around 9.8%. From the year 2018 to the end of 2020, the economy was stable again, with an average inflation of 2.6%. The aftermath of COVID-19 repercussions on inflation became apparent from the beginning of 2021 in the fields from the transport to agricultural products. Upon easing out of the COVID-19 aftermath, the global economy encountered the Russia-Ukraine war that disrupted the stability of the global economies. Azerbaijan imports higher inflation from main trade partners, which is inherent to a small economy. Starting from the end of 2022, inflation cooled due to stabilization in the global arena. Average inflation rated 10.7 percent from 2021 to the first half of 2023. Figure 1 visually highlights the fluctuations and patterns related to inflation dynamics.

Recently, CBAR has employed the VAR model to forecast two quarters for the short-term horizon and the Quarterly Projection Model (QPM) to forecast two years for the medium-term horizon. To forecast the medium range, a short-term forecast outcome is needed. Since short and medium-term forecasts serve as the foundation for monetary policy decisions of the CBAR, it is crucial to develop inflation forecast models to better understand future patterns of inflation. The paper analyzes the forecasting performance with AR, VAR, and BVAR estimation methods based on five models plus one optimal model that includes only a high-performing variable set. The monthly data used in the paper covers the period from 2003M1 to 2023M6. To compare the results of the models, we have used the pseudo-out-of-sample method using an expanding window strategy.

Figure 1: Inflation dynamics in Azerbaijan



Source: State Statistical Committee of the Republic of Azerbaijan (SSCRA)

Previously, limited number of studies have analyzed inflation rate for Azerbaijan with different estimation methods and models other than Bayesian VAR. For instance, Rahimov *et al.* (2016) explored determinants of headline inflation in Azerbaijan by utilizing the VAR model with quarterly data from 2003Q1 to 2015Q1. Their findings from impulse response analysis suggest that trade partners' inflation, fiscal policy, exchange rate and own shocks of inflation significantly explain changes in inflation. Mukhtarov *et al.* (2019) investigated the cointegration of inflation, oil price and exchange rate in Azerbaijan by employing the VECM model between 1995 and 2017. They found a statistically significant long-run relationship among those variables. They also emphasized the presence of oil price transmission into inflation through exchange rates. Additionally, the authors found that a 1% change in oil price increases inflation by 0.58%. Rahimov (2020) studied the determinants of inflation in Azerbaijan with the VAR model. The author found that the world food price index (WFPI), M2 money aggregate, nominal effective exchange rate (NEER), CPI in trading partner countries, real GDP growth, manufacturing PPI, non-oil tax and agricultural price index are the main determinants of inflation in Azerbaijan. In another research paper, a factor-augmented vector autoregressive (FAVAR) model was built by Ahmadov *et al.* (2020) to forecast inflation and output in Azerbaijan. FAVAR was employed to diminish the dimensionality problem since 77 variables were included in the model, also it mitigates omitted variable bias. However, the inference indicated that the univariate model outperforms the FAVAR model. The researchers found that a possible reason was the availability of short sample periods of data. Huseynov *et al.* (2014) studied forecasting performance of different models by using an out-of-sample method during post oil-boom period.

They estimated AR, Bayesian AR, AR-GARCH, VAR, BVAR, FAVAR and TVP-VAR starting from 2003-January to 2006-October. As a result, the naïve model showed superior performance among other sophisticated multivariate models.

Huseynov *et al.* (2014) have analyzed the forecasting ability of BVAR Litterman's prior previously with a short-term period of data. However, we have a large data sample, and we deeply focus on analyzing different models with Bayesian VAR priors. We found optimal lag and hyperparameters for mitigating shrinkage of dimensionality, comparing forecast errors for further periods with out-of-sample forecasting and expanding window strategy by taking all variables' log differences. The purpose was to obtain the stationary variables for the time series estimation techniques. Moreover, CBAR employs the VAR model for monthly short-term forecasting and makes use of the QPM model for medium-term forecasting on a quarterly basis. We employ monthly data for all models for a 24-month forecast horizon.

This paper introduced models in two structures: a category-based and incremental set of models. Model 1 acts as a base model and includes seven variables, and we subsequently add category-specific variables into models up to Model 5 to estimate the effect of those variables on inflation. Incremental models progressively integrate variables from preceding models, establishing a cumulative structure wherein each subsequent model includes the variables from the prior model along with additional variables specific to that model. Variables for both category-based and incremental model sets are described in Table 1 and Table 2, respectively.

Moreover, this paper introduced several methods for getting the final forecast outcome out of different models and estimation methods. The first approach includes forecasting all models and then selecting the same time forecast outcome based on the minimum root mean square forecast error (RMSFE) value among the models at the same time. The second approach is to select models based on the model with an average minimum RMSFE for 12 or 24 months ahead, depending on the purpose. The third approach incorporates a weighted combination of models wherein all models are forecasted monthly. Then, the forecast results of all models are multiplied by the relevant point of time weight of the models. The last approach is to forecast all models and then take a simple average of the monthly forecasts.

In general, the forecasting performance of the models in various estimation methods slightly differs from each other. If the first approach is utilized, AR, Litterman Model 2, Optimal model with Sims-Zha prior, and VAR estimation technique perform well in the first 12 months. If the second approach is utilized, the least prediction errors of average 12 months ahead, optimal model with Sims-Zha Normal Wishart prior slightly outpaces the average of the forecast horizon of the models. The third approach is the composition of forecast outcomes of all models and estimation techniques based on relative weights of forecast performance. The last approach involves calculating the simple arithmetic mean of the point forecasts derived from various models and estimation techniques.

The rest of the paper is structured as follows: Section 2 introduces Literature on Bayesian VAR used for inflation forecasting, Section 3 discusses Data and Methodology, Section 4 provides results of estimation and forecast performance of the different models, Section 5 concludes the study together with implications, limitations and leaving recommendations for further studies. In addition, supplementary materials can be found in the Appendix.

2. Literature review

Most recently, BVAR models have been used by macroeconomic institutions as the main model to predict short-term inflation with solid accuracy. However, different models and estimation methods may yield distinct inferences. We have enumerated a comprehensive review of recent literature. Most recently, BVAR models have been used by institutions as an alternative model to predict short-term inflation with strong accuracy.

Carriero et al. (2011) studies the forecasting performance of Bayesian VARs, and they assess how various specifications affect the results. They focus on a mid-size model consisting of 18 variables from the U.S. economy. Later, they determine the robustness of the outcomes by analyzing the data from Canada, France, and the U.K. To summarize the large set of results, authors found that BVAR models caused small losses on average across variables. They suggest that findings with simple methods work well and can be applied as an econometric tool for most fields of research.

Huang (2012) explored the good-fit BVAR model for inflation output growth forecasting in the Chinese economy. Quarterly data is used for estimation in the model, capturing the date range between 1998Q1 and 2005Q4. The author employed an expanding method by iterating the forecast several times to measure projection reliability for four quarters ahead. As a result of model estimation with Minnesota prior for the period between 2006 and 2012, the BVAR model reduced forecast error of inflation and output growth.

Dahem (2015) studied the forecasting performance of the standard VAR and Bayesian VAR for the Tunisian economy. A data range has been used in the models covering the 1991Q1- 2013Q4 period. The author realized that the Bayesian VECM for the mark-up model was more appropriate with lower RMSFE in comparison with the monetary model and Phillips curve for predicting cost-push inflation in Tunisia. These findings also show that forecasting using the markup model leads to a reduction of forecast errors relative to other models.

In another study, the accuracy of inflation forecast with BVAR for the Russian economy is estimated by Demeshev and Malakhovskaya (2015). The data used in the model ranges from January 1996 to April 2015, with 23 variables. They have used different sizes of models to measure whether high-dimensional model variables outperform low-dimensional models. They concluded that the model with 23 variables forecast better than a standard dimensional model with 6 or 7 variables. Also, they found that the BVAR estimation method outperforms VAR in terms of forecast reliability.

Brazdik and Franta (2017) scrutinized the forecasting performance of a small-scale mean-adjusted BVAR for the Czech economy. Quarterly data used in the model covers the 1998Q1-2016Q4 period. They compared conditional forecasts with the model and estimation method that the Czech National Bank utilizes. In conclusion, BVAR is estimated to be useful for 3-7 quarters ahead.

Vika (2018) discussed VAR and Bayesian VAR forecast ability by analyzing Albania's economy. The estimation period covers the data from 2001Q1 to 20212Q4 in an out-of-sample forecasting analysis. The scholar conducted estimation variables in log-difference in the level and percentage change for quarterly and yearly models. The results indicate that the VAR model with annual percentage changes is most preferred among others. Although Bayesian VAR lacks behind VAR model with small forecast error, the author emphasized the importance of BVAR for usage.

Öğünç (2019) studied short-term inflation forecasting models by employing quarterly data and the BVAR estimation method. The authors have utilized variables in levels or differences and estimated the accuracy of conditional and unconditional forecasts with seven BVAR models. The number of variables increases from Model 1 to Model 7. Results indicate that variables in log-difference show better results than log-levels. Among the models, they found a slight difference in forecasting accuracy up to two quarters, and conditioning helped to reduce the forecast error. The paper concluded that small and medium size BVAR models with log-differenced variables and with a normal inverted Wishart prior are favorable for short-term inflation.

Papavangjeli (2019) conducted an analysis to assess the inflation forecast reliability of the BVAR model for the Albanian economy. The data covers 16 years from 2002Q2 to 2018Q4 with nine variables from real private, financial, and external sectors. Variables in the models have been transformed into annual growth rates, except domestic and foreign interest rates. Lag selection is based on the optimal forecasting error since the information criteria function is absent. As a result, the author remarked on the success of the BVAR model among benchmark estimation methods.

Shapovalenko (2021) carried out the study related to the comparison of the BVAR and VAR QPM models that the National Bank of Ukraine utilizes for inflation forecasting in Ukraine. The data used in the research covers the date range from 2004Q1 to 2020Q1. After executing a grid search to find the optimal shrinkage value, the 0.2 value is imputed as a hyperparameter. The comparison of inflation and GDP forecasts shows that BVAR forecasts are better than QPM.

Considering the validated effectiveness and competency of BVAR estimation techniques, we utilize the BVAR estimation method, incorporating Minnesota and Sims-Zha Normal Wishart priors to develop a new model for forecasting Azerbaijani inflation. Based on the literature review, we are confident that the findings of this study will make a distinctive contribution to elevating forecast precision.

3. Data, Methods, and Methodology

3.1 Data

The monthly data used in this paper covers the period between 2003M1 and 2023M6. Data have been retrieved from the CBAR, the State Statistical Committee of the Republic of Azerbaijan (SSCRA), the Ministry of Finance of the Republic of Azerbaijan, the Energy

Information Agency (EIA), and International Monetary Fund (IMF). We have employed 17 variables, including one exogenous variable.

All variables enumerated in this study are subject to seasonal adjustment with X11 seasonally-adjustment methodology, considering monthly changes in the prices of products and services during the year. Since all variables except average loan interest for between 1 and 3 years are non-stationary in level, we transform the remaining variables into stationary by a differencing log of variables. Augmented-Dickey-Fuller tests were applied to test stationarity features of the variables before and after I (1) transformation. The log-difference transformation of needed variables rendered all variables stationary, meaning that the mean and variance of variables became constant over time. We have inserted models for both structures in Tables 1 and 2. Additional information related to the variables can be found in the table in Appendix A. Inverse roots of AR characteristic polynomial results confirm model stability, which we included in Appendix D.

3.2 Methods

We organized the models in a stepwise augmentation and incremental manner that have different objectives. The main goal of a stepwise augmentation or category-based model structure is to examine the predictive performance of different categories of economy involved in an economy. We aim to determine which of these economic category-based considerations in the Azerbaijan economy that influences the forecasting ability of the model. We intend to achieve this goal through sequential addition of variables that is taken from the varied categories of the economy, such as price indices, labor market, monetary factors, and demand in the retail sector into Model 1. Usage of this method enables us to delve into an in-depth study of the effect of specific categories and highlights critical predictors for forecasting. On the other hand, incremental models target to find out whether variable expansions affecting the predictability strength of the model in alignment with the model of Öğünç (2019). In other words, we aim to determine whether the increase in the variable set or accumulation of categories-specific factors leads to better forecasting capabilities through the gradual inclusion of new variables. The model definitions are listed below:

Model 1 – Model 1 variable set is based on Rahimov (2020) study that consists of CPI in Azerbaijan, world food price index (WFPI) calculated by IMF, weighted CPI in trade partners (TCPI), non-oil weighted nominal effective exchange rate (NEER), agricultural price index (API). In addition to Rahimov (2020), we have added *M3* money aggregate and budget expenditure into Model 1. Although Rahimov (2020) utilized *M2* in his working paper, we found a low RMSFE value when the *M3* variable is used in comparison with other money aggregates. In addition to the previous study mentioned above, we added government budget expenditure to our base model since we identified its effectiveness in lessening forecast error. The variable set is the same for both category-based and incrementally structured models.

Model 2 – Model 2 includes variables of Model 1 plus additional variables such as the industrial price index (Bańbura et al., 2010; Huang, 2012; Doan et al., 1984; Koop, 2011; and others), transport price index (Bańbura et al., 2010; Koop, 2011 and others) and manufacturing price index (Bańbura et al., 2010; Koop, 2011; and others). We assume that price change producer prices also have a contribution to inflation forecasting. As in Model 1, the variable set is the same for category-specific and incremental model structures.

Model 3 – Nominal consumer credit volume (Bańbura et al., 2010; Koop, 2011; and others) and interest rate of credits between 1 and 3 years (Huseynov et al., 2014) have been included in Model 1 as financial variables in the category-based model. Although monetary transmission is weak in Azerbaijan (Mammadov & Adigozalov, 2014), we examine the forecast performance of the variables mentioned above. For incrementally structured Model 3, we incorporated monetary variables into Model 2 to analyze the impact of the expanded number of factors from producer price indices and monetary factors on forecasting performance.

Model 4 – In Model 4, we have included into Model 1 labor market factors such as employment in non-oil sector (Öğünç, 2019; Bańbura et al., 2010; Koop, 2011; Papavangjeli (2019); and others) and nominal average wage (Bańbura et al., 2010; Papavangjeli (2019); and others) variable set for the economic category-based structured model. The purpose of this model is to find the effect of labor market activity on inflation. For the incrementally designed model group, Model 4 is developed by adding the above-mentioned labor market variables into Model 3 to examine the aggregate effect of factors from different categories of economy we discussed till this model.

Model 5 – In Model 5, demand factors have been added to Model 1. Although, as demand factors, real GDP growth and output could be used, since we structured monthly models, we employed the sum of catering and paid services, and trade turnover as a representative of demand in the economy to study the significance of demand determinants on inflation. For incrementally organized Model 5, demand factors have been integrated into aggregate Model 4 of the incremental model group. This grants a comprehensive study of the factors from all categories of economy we assumed have the contribution to determining the future of inflation dynamics.

Optimal Model – Additionally, this study suggested the optimal model with the minimum RMSFE value by examining Model 5 within an incrementally organized group of models. In this process, we eliminated the variables that diminish forecast performance from Model 5 containing all study variables. As a result, we are left with all Model 1 variables, industrial and manufacturing producer price indices from Model 2, average loan interest of banks for 1 and 3 years from Model 3, and the number of hired workers from Model 4. None of the variables from Model 5 is selected due to their diminishing effect on the forecast performance of the inflation rate.

To all models in both structured models set, we have included the log-difference of Brent oil price in its 3rd lag as an exogenous variable since it has a significant minimizing effect on forecast errors of the whole models in the 3rd lag. We have described models for both structures in Tables 1 and 2 in briefly.

Table 1: Economic category-based structured models

Variables	AR	Optimal model	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
CPI	*	*	*	*	*	*	*	Base model	M1+ producer and service price indices	M1 + Monetary variables	M1 + Labour market	M1 + Demand factors
World food prices		*	*	*	*	*	*					
CPI in trading partners		*	*	*	*	*	*					
Non-oil import weighted NEER		*	*	*	*	*	*					
Agricultural PPI		*	*	*	*	*	*					
Budget expenditure		*	*	*	*	*	*					
M3		*	*	*	*	*	*					
Industrial PPI		*		*								
Transport PPI				*								
Manufacturing PPI		*		*								
Consumer credit volume					*							
1–3-year average loan interest		*			*							
Employment		*				*						
Nominal Wage						*						
Catering and paid services							*					
Trade turnover							*					
Oil price - Exogenous variable		*	*	*	*	*	*					

Source: Author's construction

Table 1 represents the category-based model sets. Model 1 contains core variables that previous studies and our initial analysis have found significant explanatory models with different estimation methods. Other models add up variables to Model 1 set-by-classifications of the economy.

Table 2: Incrementally structured models

Variables	AR	Optimal model	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
CPI	*	*	*	*	*	*	*	Base model = M1	M1+ producer and service price indices	M2 + Monetary variables	M3 + Labour market	M4 + Demand factors
World food prices		*	*	*	*	*	*					
CPI in trading partners		*	*	*	*	*	*					
Non-oil import weighted NEER		*	*	*	*	*	*					
Agricultural PPI		*	*	*	*	*	*					
Budget expenditure		*	*	*	*	*	*					
M3		*	*	*	*	*	*					
Industrial PPI		*		*	*	*	*					
Transport PPI				*	*	*	*					
Manufacturing PPI		*		*	*	*	*					
Consumer credit volume					*	*	*					
1–3-year average loan interest		*			*	*	*					
Employment		*				*	*					
Nominal Wage						*	*					
Catering and paid services							*					
Trade turnover							*					
Oil price - Exogenous variable		*	*	*	*	*	*					

Source: Author's construction

Table 2 represents the structure of incremental model sets. Model 1 contains core variables that previous studies and our initial analysis have found significant explanatory models with different estimation methods. Other models add category-specific variables incrementally into the previous model.

3.3 Methodology

Standard Vector Autoregressive (VAR) estimation methods allow us to capture dynamic interrelationships among a set of endogenous variables.

$$\boldsymbol{y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots \beta_p y_{t-p} + E x_t + \varepsilon_t}, \textbf{ (1)}$$

$$\boldsymbol{\varepsilon_t \sim N(0, \Sigma)} \textbf{ (2)}$$

where $\boldsymbol{y_t}$ denotes the matrix encompassing n endogenous variables, presenting a comprehensive view of the interrelated stationary variables; $\boldsymbol{x_t}$ represents a vector of exogenous variables, incorporating both the constant term and additional exogenous regressors to enrich the model's explanatory power. To define coefficients $\boldsymbol{\beta}$ stands as a coefficient of endogenous variables and ***E*** as a coefficient of exogenous variables. Subscript ***p*** and ***t*** denote lag length and indicate the specific period at which the model is being applied or estimated, respectively.

To generalize the above equation:

$$\boldsymbol{Y_t = X_t A + \varepsilon_t} \textbf{ (3)}$$

Shortly, we define all coefficients collectively by ***A***, endogenous and exogenous variables as ***X*** at time $\boldsymbol{t}$.

To write the above equation in a vectorized form:

$$\boldsymbol{y = (l_n \otimes X) B + \varepsilon} \textbf{ (4)}$$

where ***y*** is vec(*Y*), ***B*** is vec (A), and $\boldsymbol{\varepsilon}$ is vec(∈).

VARs have been widely used for forecasting over the past three decades. The existing literature on this subject stresses the point that unrestricted VARs encounter over-parametrization problems as the number of explanatory variables increases (see Ciccarelli and Rebucci, 2003; De Mol et al., 2008) which may cause poor estimation and forecasting performance. This problem is rooted in the fact that several parameters grow non-linearly as the variable size increases (Demeshev and Malakhovskaya, 2015; Papavangjeli, 2019). In other words, available data may not always involve all information to analyze, meaning that there can be trivial parameters that may harm forecasting performance. The prior distribution acts as a shield, restricting parameters from becoming zero by inputting information we determine (Sevinç & Ergün, 2009). To transform the complex VAR model into parsimony, either we can apply restrictions on some parameters where we assume the relationship is weak, or we can use other estimation methods to replace standard VAR. Bayesian VAR is one alternative for restricting over-parametrization by imposing restrictions via prior beliefs, for which we have employed two of them: Litterman/Minnesota and Sims-Zha Normal Wishart priors. The fundamental concept of the BVAR estimation technique is to restrict over-parametrization by imposing prior beliefs. Hence, it reduces parameter uncertainty and rises the forecast performance of the models.

In alignment with Litterman (1986), the standard deviation of the prior distribution for lag $\boldsymbol{l}$ of the variable $\boldsymbol{j}$ in equation $\boldsymbol{i}$ is given by:

$$\boldsymbol{y_t = A_0 + y_{t-1} + \varepsilon_t} \textbf{ (5)}$$

$$\boldsymbol{S(i,j,l)} = \frac{[\boldsymbol{\gamma g(l) f(i,j)}]\boldsymbol{s_i}}{\boldsymbol{s_j}} \quad \textbf{(6)}$$

$\boldsymbol{s_i}$ signifies the standard error of univariate regression on the equation $\boldsymbol{i}$. The ration $\frac{s_i}{s_j}$ stands for correction of the standard deviation of the variables. $\boldsymbol{\gamma}$ denotes the overall tightness and is also the standard deviation on the first own lag. $\boldsymbol{g(l)}$ determines the tightness of the lag $\boldsymbol{l}$ relative to lag one and can be type of harmonic or geometric with a decay factor $\boldsymbol{d}$ of one or two. It tightens the prior on increasing lags. $\boldsymbol{g(l)}$ decays harmonically with $\boldsymbol{g(l)} = \boldsymbol{l^{-d}}$. Geometric type of $\boldsymbol{g(l)}$ tends to get tight very fast. The parameter $\boldsymbol{f(i,j)}$ represents tightness of variable $\boldsymbol{j}$ in equation $\boldsymbol{i}$ relative to variable $\boldsymbol{i}$ with the relative tightness coefficient $\boldsymbol{w}$. For deterministic variables the priors are uninformative. In the literature, overall tightness $\boldsymbol{\gamma}$, lag decay factor $\boldsymbol{d}$ and weight parameter $\boldsymbol{f(i,j)}$ are called as hyperparameters.

Following Blake & Mumtaz (2012) we can write Normal Wishart prior probability distribution as below:

$$\boldsymbol{p(B|\Sigma_\varepsilon) \sim N(\widehat{B}_0, \Sigma_\varepsilon \otimes H)} \quad \textbf{(7)}$$

$$\boldsymbol{P(\Sigma_\varepsilon) \sim Iw(\bar{s}, \delta)} \quad \textbf{(8)}$$

$\boldsymbol{\widehat{B}_0}$ is defined as prior probability distribution, the matrix $\boldsymbol{H}$ is a diagonal matrix that are for coefficients on lags can be defined as $\left(\frac{\lambda_0\lambda_1}{l^{\lambda_3}\sigma_i}\right)^2$ and for constant can be defined as $(\lambda_0\lambda_4)^2$, $\boldsymbol{\bar{s}}$ is defined as N x N diagonal matrix with diagonal elemement with is as below:

$$\boldsymbol{\bar{s}} = \left(\frac{\sigma_i}{\lambda_0}\right)^2 \textbf{(9)}$$

$\boldsymbol{\bar{s}}$ can be re-written in a matrix form:

$$\boldsymbol{\bar{s}} = \begin{pmatrix} \left(\frac{\sigma_1}{\lambda_0}\right)^2 & \boldsymbol{0} \\ \boldsymbol{0} & \left(\frac{\sigma_2}{\lambda_0}\right)^2 \end{pmatrix} \textbf{(10)}$$

Hyperparameters that make up the diagonal elements can be defined as following: $\boldsymbol{\lambda_0}$ which is overall tightness of the covariance matrix, $\boldsymbol{\lambda_1}$ is the overall tightness of the priors on the first lag, $\boldsymbol{\lambda_3}$ a is lag decay that controls the degree to which coefficients on lags higher than 1 are likely to be zero, $\boldsymbol{\lambda_4}$ is the control variable on constant. Further procedure of the equation can be found in Blake & Mumtaz (2012) research paper.

Some studies have employed standard values accepted by Minnesota hyperparameter values. On the contrary, to find a proper hyperparameter that strengthens forecasting accuracy, we conduct a grid search in line with Giannone et al. (2012), Dieppe et al. (2016), Papavangjeli (2019), Shapovalenko (2021) to find the appropriate hyperparameter values which simultaneously it rises marginal likelihood for the models (Dieppe et al., 2016). We check the overall tightness parameter from 0 to 1, incrementing step-by-step with an increase of 0.05 and lag decay from 1 to 4. The range of lag decay implemented in this study captures the short-term dynamics of the variables that typically occur within the first few periods while maintaining parsimonious framework of the analysis. Residual covariance tightness is treated as constant in Litterman’s prior (Carriero et al., 2011), while for Sims-Zha (NW), we input values from 0 to 1. Inversely, cross-variable weight is fixed in Sim-Zha (NW), while Litterman’s prior accepts value ranges between 0 and 1. Consequently, with the increase in the number of variables, especially in the incrementally designed model set, we observed that hyperparameter values needed to be increased close to 1 for the absence of autocorrelation and improved forecast performance, which is in line with the results of Bańbura et al. (2010).

Lag selection is the fundamental issue for VAR models that higher lags reduce the degrees of freedom, while lower lags may not capture the inter-temporal relationship. Also, a lower count of lags may raise concerns over the serial correlation problem (Lack, 2006). This underscores the necessity of selecting a proper lag length that ensures effective forecasting performance. In line with Giannone et al. (2015), Papavangjeli (2019), and Vika (2021), employing RMSFE for lag selection assists us in the identification of optimal lag length with minimized reduced forecast error. Our findings suggest that, regardless of the estimation method, all models consistently manifest low prediction errors within three lags. The importance of the lag length is also emphasized by Carriero et al. (2011) that adoption of shorter lags usually results in improved forecast accuracy. The choice of 3-lag selection leads to the removal of serial correlation across the complete set of models for all estimation techniques in this study. Thus, more than three lags autocorrelation issue becomes troublesome, while less than three lag order selection deliver high forecast error. All hyperparameter values and lag lengths of models estimated in this study, shaped by the forecast performance of models and estimation methods through thousands of repeated iterations, are summarized in Table 3.

Table 3: Hyperparameter and lag specification

	VAR	Sims-Zha (NW)	Litterman/Minnesota
AR coefficient	-	0	0
Residual covariance tightness	-	1	-
Overall tightness	-	0.9	0.9
Relative cross-variable weight	-	-	1
Lag decay	-	1	1
Lag length	3	3	3

Source: Author’s calculation.
Notes: The table contains the significant information we used during estimation of 3 estimation techniques and models, with all inputs satisfying the requirements of ordinary least squares.

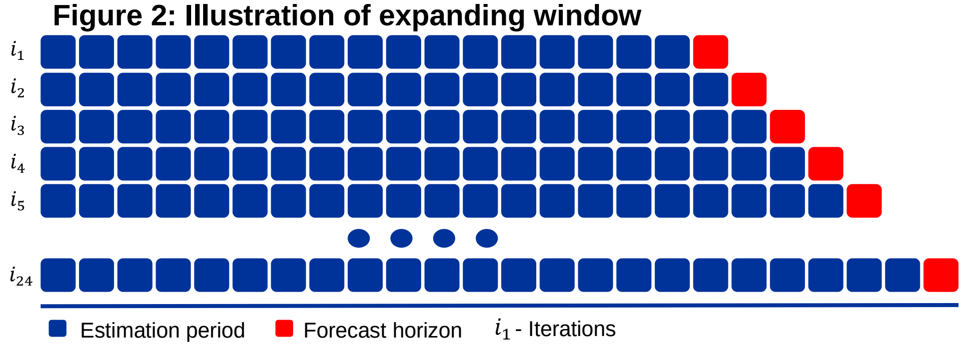
Following the determination of appropriate inputs above, we have conducted several diagnostics tests. After successive test results, we step into forecasting the performance of the models and estimation techniques to reach our objective. Using the expanding strategy method, we conducted model estimations between January 2003 and June 2020 by utilizing the out-of-sample technique, expanding the model 24 iterations with a forecast of 24 months ahead, covering realized historical data until May 2023. Expanding strategy can be a useful approach through a periodic increase in the number of observations by one month in each iteration, leading to accumulating data which may yield a better-fit model and improved forecasting accuracy. In contrast, the rolling window strategy has a fixed range of observations, and in each iteration, the range shifts one period forward from both sides, and the number of observations remains the same. Hence, the expanding strategy is deemed efficient due to the preceding statement. In alignment with the equation suggested by Papavangjeli (2019), we conduct root mean square forecast error (RMSFE) analysis with an expanding strategy represented below:

$$RMSFE^h = \sqrt{\frac{1}{T-h+1} \sum_{i=\mathrm{R}}^{T-h} (y - \hat{y})^2_{t+h}} \quad \textbf{(11)}$$

where $\boldsymbol{y} - \boldsymbol{\hat{y}}$ is the difference of actual values and predicted values for the relative period; ***T*** represents the total span of the entire dataset, separated into in-sample ***R*** and out-of-sample period ***P***; ***h*** denotes the forecast horizon; ***t*** is the timepoint forecast. We conduct out-of-sample forecast from ***R+h*** to ***T****.* We have portrayed RMSFE assessment with expanding window strategy in Figure 2.

To examine the effectiveness of our models, we compare them with an autoregressive (AR) model. The corresponding AR model, simple and parsimonious, is the baseline forecast for evaluation as to how much information can be predicted by the developed model(s). We employed conventional VAR with the same variables we used in Model 1 for additional evaluation of the models with the Bayesian estimation method. This strengthens the interpretation ability of the inferences and establishes a pathway of comprehensive assessment between our proposed model and existing benchmarks within modern econometric literature.

Figure 2: Illustration of expanding window



Source: Author's construction

Notes: The figure illustrates the assessment procedure of forecast error of the models. The training time interval is identified with blue boxes, and the out-of-sample period is identified with red boxes, lasting for 24 months. Subscript *i* represents iterations from 1 to 24 utilized for expanding strategy.

In our study, we applied a 4-fold approach to improve the precision and reliability of inflation forecasting with different approaches. In the first approach, we assessed individual models' accuracy for 24 months ahead using RMSFEs; those with lower values in specific periods were considered as potential candidates to use as forecast. To optimize the accuracy of models in the prediction of inflation dynamics, all models can be estimated and forecasted, and later predicted results can be selected according to lower RMSFE values in the relative periods.

Second, we built an average RMSFE value table for 12 and 24 months of all models. Based on the results derived from the first approach, we depict and compare the inferences of the models. Pursuing this approach, based on the lower average RMSFE values for 12 or 24 months, depending on the horizon forecast purpose, the concrete model can be selected for forecasting.

As a third strategy to get optimal forecast outcomes, we introduced a weighted average approach to take advantage of the characteristics of various models and produce more accurate inflation projections. Acknowledging that models may perform well in varying economic environments, the weighted combination approach allows us to blend information from various models. Aiolfi et al. (2010) found that combination forecasts outperform model forecasts from different models. The combination approach made it easier to generate inflation forecasts by assigning weights to each model in proportion to their historical RMSFE performance in the same time point for 24 months. Mathematically, the weighted combination method is expressed as follows:

Let's denote the RMSFE for models in this study:

$$\boldsymbol{RMSFE_i} \text{ where } i = 1, 2, 3, 4 \dots 19 \text{ models}$$

The weight assigned to each model is inversely proportional to its RMFSE:

$$\boldsymbol{w_i} = \frac{1}{RMSFE_i} \quad \textbf{(12)}$$

$$\boldsymbol{w^*_i} = \frac{w_i}{\Sigma_{t=1}^{24}(w_t)} \quad \textbf{(13)}$$

$$\boldsymbol{CF_t} = \Sigma_{t=1}^{19}\, \boldsymbol{w^*_{i,\,t}} \times \boldsymbol{F_{i,t}} \quad \textbf{(14)}$$

where $\boldsymbol{w}$ represents weights of models derived based on RMSFE values; $\boldsymbol{w^*}$ is the normalized weight introduced in equation 3 that contains the sum of the inverse **RMSFE** values to equalize to 1; ***F*** or denotes forecast outcomes of models; ***CF*** represents combined forecast outcomes by sum of multiplying normalized weights of models to model outcomes in the timepoint. The subscripts ***i*** and ***t*** symbolize the model and forecast timepoint, respectively.

This method provides a strategy in which the effect of each model is determined by its historical predictive accuracy. Models with lower RMSFEs have higher weights, which means that they have a more substantial effect on the composite forecast.

4. Results and discussion

Before forecasting, the stability test and impulse response were assessed. From the results we included in the Appendix, all models of the three estimation techniques communicate that all models are stable. Impulse responses also show that responses of variables to shocks are in line with the theory.

Our empirical analysis included a wide range of forecasting models, starting from classical auto-regressive (AR) and vector auto-regressive (VAR), going through to Bayesian VARs with Minnesota and Sims-Zha normal Wishart priors to maximize inflation forecast precision. All models have been estimated from 2003M1 to 2019M6, then forecasted for 24 periods ahead by iterating 24 times using an expanding strategy. For the sake of saving space, we compiled the detailed RMSFE value in Appendix B1 and B2 parts of the paper. Regarding the discussion in the methodology part about four strategies to achieve more accurate forecast outcomes, we are going to present analysis results and interpretation in sequence. To start with the first approach, inferences suggest that all models, including AR and VAR, which we deemed as benchmark models, play a significant role in forming forecasting dynamics. To interpret the findings presented in Table 4, both incremental and categorical models have the same set of models that exhibit superior performance in the first 12 months. It is also worth mentioning that forecast performance difference is negligible in the same forecast horizon of incremental and categorical models between 12 and 24 forecast spans. Elaborating on the forecast performance of the first 12 months further, the naïve model outperforms multivariate models consecutively for the first three months. In later periods till the end of the forecast horizon, AR model accuracy faded among other models. Thus, Model 2 of Minnesota prior takes the 4th and 5th months. Sims-Zha's (NW) optimal model captures the 6th, 7th and 8th months of forecast interval with the values approaching the lower bound of RMSFEs among the whole projected time horizon. VAR optimal model captures the further periods to complete one year ahead forecast. Since short-term forecast outcome is used for medium and long-term forecast models, it is needed to report in a quarterly basis, too. Thus, the AR and Sims-Zha NW Optimal Models perform well for the 1st and 2nd quarters, respectively, while for the 3rd and 4th quarters, the VAR Optimal Model gains significance as a contributor. Later periods change depending on the model structure's characteristics, which can be seen in Table 4. Overall, there is no significant forecast error difference between the 12 and 24-month forecast intervals. However, incrementally structured models seem slightly better-performing models.

Table 4: Models with minimum RMSFE values of incremental and category-based models

Forecast horizon	Incremental models with min RMSFEs	Min RMSFE	Category-based models with min RMSFEs	Min RMSFE
1M	AR Model	0.35	AR Model	0.35
2M	AR Model	0.43	AR Model	0.43
3M	AR Model	0.41	AR Model	0.41
4M	Litterman Model 2	0.40	Litterman Model 2	0.40
5M	Litterman Model 2	0.39	Litterman Model 2	0.39
6M	Sims-Zha (NW) Optimal Model	0.37	Sims-Zha (NW) Optimal Model	0.37
7M	Sims-Zha (NW) Optimal Model	0.35	Sims-Zha (NW) Optimal Model	0.35
8M	Sims-Zha (NW) Optimal Model	0.36	Sims-Zha (NW) Optimal Model	0.36
9M	VAR Model Optimal	0.37	VAR Model Optimal	0.37
10M	VAR Model Optimal	0.38	VAR Model Optimal	0.38
11M	VAR Model Optimal	0.40	VAR Model Optimal	0.40
12M	VAR Model Optimal	0.43	VAR Model Optimal	0.43
13M	Litterman Model 4	0.42	VAR Model Optimal	0.44
14M	Litterman Model 4	0.41	VAR Model Optimal	0.43
15M	VAR Model 3	0.44	VAR Model 2	0.44
16M	Litterman Model 4	0.56	Sims-Zha Model 2	0.58
17M	Litterman Model 4	0.60	VAR Model Optimal	0.62
18M	Litterman Model 4	0.60	Sims-Zha Model 2	0.61
19M	VAR Model 4	0.58	Sims-Zha Model 2	0.60
20M	VAR Model 4	0.58	VAR Model Optimal	0.61
21M	VAR Model 4	0.58	Sims-Zha Model 2	0.60
22M	VAR Model 4	0.57	Sims-Zha Model 2	0.60
23M	VAR Model 4	0.57	Sims-Zha Model 2	0.60
24M	VAR Model 4	0.57	Litterman Model 2	0.60
12 months average	-	0.39	-	0.39
24 months average	-	0.46	-	0.47

Source: Author's calculations.
Note: The table provides information on the RMSFE of the models utilizing various estimation methods, spanning 12- and 24-month averages. Models with lower RMSFE have been listed in the table. Cells shaded with deep green color highlight lower forecast error, while those in a lighter tone indicate higher forecast error.

Employing a secondary approach for model choice to seek more accurate prediction, we have combined average forecast errors in Table 5 for 12 and 24 months long to be used depending on forecast horizon purpose. On average, optimal models yield better outcomes relative to other models, including both incremental and category-based models, and Bayesian VAR with Sims-Zha Normal Wishart marginally demonstrates superiority out of 3 optimal models. Model 3, Model 4 and Model 5, distinguished by starred annotations, highlight the outcomes of models from structural models' basket. The difference becomes noticeable upon scrutinizing the model sets in Tables 1 and 2, which show that variable sets are different in incremental and category-specific structured groups due to purpose. Notably, while Model 1 and Model 2 exhibit uniformity, the variable sets vary from Model 3 to Model 5. Therefore, forecast outcomes for Model 1, Model 2 and optimal models of the three estimations methods are the same for two different structured model sets.

Table 5: Average forecast performance of the models

Models	**Incremental models**		**Category-based models**	
	12 months average	**24 months average**	**12 months average**	**24 months average**
AR Model	0.46	0.54	0.46	0.54
VAR Model 1	0.51	0.57	0.51	0.57
VAR Model 2	0.49	0.53	0.49	0.53
VAR Model 3*	0.50	0.53	0.54	0.59
VAR Model 4*	0.56	0.55	0.50	0.57
VAR Model 5*	0.58	0.61	0.53	0.59
VAR Model Optimal	0.44	0.50	0.44	0.50
Litterman Model 1	0.50	0.57	0.50	0.57
Litterman Model 2	0.46	0.52	0.46	0.52
Litterman Model 3*	0.48	0.52	0.51	0.57
Litterman Model 4*	0.52	0.53	0.48	0.55
Litterman Model 5*	0.54	0.56	0.50	0.58
Litterman Optimal Model	0.44	0.51	0.44	0.51
Sims-Zha Model 1	0.50	0.57	0.50	0.57
Sims-Zha Model 2	0.47	0.52	0.47	0.52
Sims-Zha Model 3*	0.48	0.53	0.52	0.57
Sims-Zha Model 4*	0.53	0.54	0.53	0.58
Sims-Zha Model 5*	0.54	0.57	0.51	0.57
Sims-Zha Optimal Model	0.43	0.51	0.43	0.51

Source: Author's calculations.
Note: The table provides information on the RMSFE of the models utilizing various estimation methods, spanning 12- and 24-month averages. The asterisk (*) denotes that model forecast errors are different depending on the model's structure. Models shaded with deep green color highlight lower forecast error, while those in a lighter tone indicate higher forecast error.

We can observe that from Model 1 to Model 5 of the three estimation techniques, the trend of average prediction error is rising, except for Model 2, which gains supremacy in accuracy. While mean forecast errors slightly surge from Model 3 to Model 5 of BVAR priors in incremental models set, the rate of ascent of those in VAR models barely outperforms BVAR models. This phenomenon can arise as a cause of overparameterization issues in VAR. The difference between BVAR and VAR techniques is more visible if the comparison is conducted through 24-month average RMSFE values. In the category-based models' group, the trend of RMSFE values almost remains flat due to the modest change in the variable count of models from Model 1 to Model 5. To compare forecast errors with Table 4 and Table 5, models with minimum RMSFE values, of which the average is 0.39, have better performance in comparison with average forecast performance models, which the model with minimum RMSFE is 0.43. More detailed figures about out-of-sample forecast errors can be found in Appendix B1 and B2.

As a third approach for forecasting 1 or 2 years ahead, depending on the purpose, we suggest a combination weighted method. We calculated combination forecasts based on out-of-sample forecast errors of all models. Weights of relevant models for both structures have been inserted in the appendix part.

As mentioned in the methodology part, all models have different characteristics, from the estimation method to the number of factors employed in the models. Therefore, acquiring all information from all models based on their performance weight can also offer composite forecast results. Appendix C1 and C2 tables are designed to use a weighted combination forecast of all models. Applying the equations we presented in the methodology part, those weights can be utilized to find timepoint forecasts by aggregating products obtained by multiplying the timepoint forecasts of all models with their relevant weights. By repeating this formula, multistep forecasts can be achieved. Approach 3 allows us to involve the contribution of all models and estimation techniques.

Appendix C1 here

Appendix C2 here

In a final approach, we suggested taking a simple mean average of outcomes from all models we analyzed in this study. However, a simple average approach may cause a diverge in the ultimate outcome from the effective forecast result. As confirmed by Aiolfi et al. (2010), it would be better to utilize the combination method, which blends all prediction outcomes of the models with different characteristics.

5. Conclusion

Our goal is twofold in this paper. We present short-term inflation forecasting models with the BVAR estimation method that most recently gained popularity for its strengths in alleviating the overparameterization problem inherent to standard VAR and suggest forecasting strategies to utilize the models. This implies that the inclusion of more variables that explain the focus variable does not pose an issue, unlike the VAR estimation technique. Therefore, we have built a base model with seven endogenous variables and one exogenous variable that are the main explanatory variables of inflation in Azerbaijan and supplementary models that capture factors from different parts of the economy. We designed this set of models in an economic category-focused manner. The second type of model set consists of incrementally organized models. The variables of category-specific models, starting from Model 2 to Model 5, accumulated their variables to their preceding models. Model 5 of the incrementally structured model consists of 16 endogenous variables and one exogenous variable. These two ways of model structuring allow us to assess, respectively, how the factors associated with various categories of economy and the accumulation of these factors contribute to diminishing forecast error. We have estimated these two groups of models with AR, VAR, Minnesota, and Sims-Zha Normal Wishart priors of BVAR. AR and VAR estimation techniques are employed to compare the forecast performance of the priors of the BVAR estimation method. Afterwards, we built an alternative model, labelled as the Optimal model, by preserving only significant variables that lessen forecast error out of the whole variable set.

Quality comparison of structure, model and estimation methods are assessed through forecast accuracy gauge – RMSFE. Utilizing the expanding window strategy, we have forecasted 24 months ahead, expanding step-by-step by 24 times to measure the average forecast error of each month. After conducting the analysis, we derived numerous interpretations with respect to structure, model, and estimation methods. Later, we yielded four strategies for obtaining the ultimate forecast figure. The estimated results indicate that all models involving the benchmark AR model can have a contribution to the models. Accordingly, if approach one is accepted for forecasting strategy, the AR model significantly outperforms the VAR and BVAR models for the first three months. The following two months are marked by Litterman Model 2 forecasting performance, which slightly differs among the three multivariate estimation methods. The remaining seven months are dominated by the optimal model we developed with Sims-Zha prior of BVAR and standard VAR to complete the 12-month horizon. For the ^{1s}t and 2^{nd} quarters of the forecast horizon, the AR and Sims-Zha NW Optimal models show better performance, while the 3^{rd} and 4^{th} quarters are secured by the VAR Optimal model. In the context of average forecast error, which we highlighted as 2^{nd} approach for forecasting strategy among both economic category-based and incrementally structured model sets for 12 months forecast horizon BVAR Sims-Zha (NW) Optimal model and for 24 months forecast horizon standard VAR Optimal model outpaces the competitive models. Meanwhile, incrementally structured models are built in alignment with Öğünç (2019) to compare the performance of the complex model with the simple one and the models with a small number of variables perform better than those of many, which is consistent with the same study. The initial two approaches focus on evaluating the performance of individual models independently; the third and fourth approaches encompass all the forecast contributions of all models together with the estimation methods we utilized in this study. Thus, the third approach blends outcomes regarding the weighted average of forecast accuracy of the models and estimation techniques; 4^{th} approach encompasses only the simple average of the outcome of those. By referring to the prior literature, we suggested employing a weighted combination forecast technique to get a reliable ultimate forecast outcome.

The main limitation we have encountered is the need for more data available for some variables. The date range for some variables is lower than the maximum range, which could be the main reason for the poor model performance of some models. Based on the forecast error outcomes, we can conclude that Bayesian VARs can be a valuable tool for forecasting when following the weighted combination method approach, we discussed above. Further studies may focus on modelling inflation via time-varying parameter Bayesian VAR techniques.

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Appendix A: Description of data set

Variables - in levels	Source	Status in level	Data range	Data description
CPI	CBAR	Non-stationary	2003M1-Present	Consumer price index
World food prices	IMF	Non-stationary	2003M1-Present	World food and beverages index of IMF
CPI in trading partners	CBAR	Non-stationary	2003M1-Present	CPI in trading partners of Azerbaijan based on non-oil import weight
N.o.i.w NEER	CBAR	Non-stationary	2003M1-Present	Nominal effective exchange with trading partners of Azerbaijan based on non-oil import weight
Agricultural PPI	CBAR	Non-stationary	2003M1-Present	Price index of agricultural products
Budget expenditure	MFRA	Non-stationary	2003M1-Present	Monthly nominal budget expenditure of Azerbaijan
M3	CBAR	Non-stationary	2003M1-Present	M3 nominal money aggregate
Industrial PPI	SSCRA	Non-stationary	2003M1-Present	Industrial producer price index
Transport PPI	SSCRA	Non-stationary	2005M1-Present	Transport producer price index
Manufacturing PPI	SSCRA	Non-stationary	2003M1-Present	Manufacturing producer price index
Consumer credit	CBAR	Non-stationary	2006M1-Present	Consumer credits in nominal volume
1-3 year interest rate	CBAR	Stationary	2005M1-Present	1-3 year weighted average loan interest rate offered by banks
Employment	SSCRA	Non-stationary	2003M1-Present	Number of hired employees
Nominal Wage	SSCRA	Non-stationary	2003M1-Present	Average nominal wage in the country
Catering and paid services	SSCRA	Non-stationary	2003M1-Present	Sum of nominal volume expenditure on catering and paid services
Trade turnover	SSCRA	Non-stationary	2003M1-Present	Volume of nominal trade turnover
Brent oil - Exogenous variable	EIA	Non-stationary	2003M1-Present	Average monthly nominal Brent oil price

Source: Constructed by author

Appendix B1

endix B1: RMSFEs of incremental models

Models	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M	11M	12M	13M	14M	15M	16M	17M	18M	19M	20M	21M	22M	23M	24M	Average of 12 months	Average of 24 months
AR Model	0.35	0.43	0.41	0.43	0.46	0.47	0.48	0.50	0.48	0.49	0.51	0.52	0.53	0.52	0.53	0.66	0.66	0.66	0.66	0.66	0.65	0.64	0.64	0.65	0.46	0.54
VAR Model 1	0.48	0.56	0.53	0.42	0.41	0.44	0.48	0.53	0.54	0.55	0.55	0.57	0.59	0.58	0.57	0.66	0.67	0.67	0.67	0.66	0.66	0.66	0.65	0.65	0.51	0.57
VAR Model 2	0.73	0.61	0.60	0.41	0.41	0.45	0.43	0.43	0.41	0.43	0.45	0.48	0.48	0.43	0.44	0.59	0.62	0.62	0.61	0.61	0.61	0.60	0.60	0.60	0.49	0.53
VAR Model 3	0.76	0.66	0.67	0.48	0.44	0.46	0.41	0.40	0.40	0.41	0.42	0.44	0.45	0.42	0.44	0.58	0.62	0.62	0.61	0.61	0.61	0.60	0.60	0.60	0.50	0.53
VAR Model 4	0.73	0.71	0.72	0.61	0.56	0.64	0.52	0.49	0.44	0.45	0.44	0.46	0.45	0.43	0.46	0.58	0.61	0.61	0.58	0.58	0.58	0.57	0.57	0.57	0.56	0.55
VAR Model 5	0.80	0.74	0.66	0.60	0.65	0.67	0.49	0.44	0.39	0.46	0.50	0.54	0.52	0.49	0.57	0.66	0.73	0.74	0.69	0.69	0.69	0.64	0.65	0.67	0.58	0.61
VAR Model Optimal	0.61	0.61	0.58	0.43	0.40	0.39	0.36	0.37	0.37	0.38	0.40	0.43	0.44	0.43	0.44	0.59	0.62	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.44	0.50
Litterman Model 1	0.47	0.56	0.53	0.44	0.42	0.44	0.48	0.52	0.53	0.54	0.54	0.57	0.58	0.58	0.55	0.64	0.67	0.66	0.67	0.66	0.65	0.63	0.65	0.65	0.50	0.57
Litterman Model 2	0.68	0.59	0.55	0.40	0.39	0.41	0.41	0.40	0.41	0.43	0.42	0.46	0.47	0.46	0.47	0.59	0.63	0.62	0.63	0.61	0.61	0.62	0.61	0.60	0.46	0.52
Litterman Model 3	0.69	0.63	0.60	0.47	0.46	0.43	0.40	0.41	0.42	0.41	0.41	0.43	0.43	0.43	0.45	0.58	0.62	0.61	0.60	0.61	0.59	0.60	0.63	0.61	0.48	0.52
Litterman Model 4	0.66	0.67	0.65	0.56	0.51	0.54	0.48	0.47	0.44	0.44	0.42	0.43	0.42	0.41	0.44	0.56	0.60	0.60	0.59	0.59	0.58	0.60	0.58	0.58	0.52	0.53
Litterman Model 5	0.71	0.69	0.60	0.53	0.57	0.59	0.47	0.45	0.43	0.45	0.47	0.48	0.46	0.45	0.49	0.62	0.68	0.65	0.64	0.65	0.64	0.61	0.62	0.60	0.54	0.56
Litterman Optimal Model	0.58	0.59	0.55	0.41	0.39	0.38	0.37	0.37	0.38	0.39	0.40	0.44	0.46	0.46	0.46	0.62	0.64	0.63	0.62	0.62	0.64	0.63	0.62	0.63	0.44	0.51
Sims-Zha Model 1	0.47	0.56	0.53	0.43	0.42	0.44	0.48	0.53	0.54	0.55	0.55	0.57	0.58	0.58	0.57	0.65	0.66	0.65	0.66	0.66	0.65	0.64	0.64	0.65	0.50	0.57
Sims-Zha Model 2	0.69	0.60	0.56	0.40	0.40	0.41	0.40	0.41	0.41	0.43	0.44	0.46	0.47	0.44	0.45	0.58	0.62	0.61	0.60	0.61	0.60	0.60	0.60	0.60	0.47	0.52
Sims-Zha Model 3	0.70	0.63	0.62	0.46	0.44	0.44	0.40	0.40	0.41	0.42	0.42	0.45	0.44	0.45	0.45	0.60	0.62	0.63	0.62	0.62	0.62	0.62	0.62	0.62	0.48	0.53
Sims-Zha Model 4	0.67	0.67	0.67	0.57	0.52	0.56	0.49	0.47	0.44	0.44	0.43	0.45	0.43	0.44	0.46	0.58	0.61	0.62	0.59	0.60	0.59	0.58	0.58	0.58	0.53	0.54
Sims-Zha Model 5	0.73	0.69	0.63	0.54	0.58	0.60	0.48	0.45	0.42	0.46	0.46	0.48	0.47	0.46	0.50	0.63	0.68	0.68	0.67	0.66	0.65	0.62	0.63	0.64	0.54	0.57
Sims-Zha Normal Wishart Optimal Model	0.58	0.60	0.55	0.41	0.40	0.37	0.35	0.36	0.37	0.38	0.40	0.43	0.44	0.44	0.47	0.61	0.65	0.62	0.62	0.62	0.61	0.62	0.61	0.62	0.43	0.51
Minimum RMSFEs of Models	0.35	0.43	0.41	0.40	0.39	0.37	0.35	0.36	0.37	0.38	0.40	0.43	0.42	0.41	0.44	0.56	0.60	0.60	0.58	0.58	0.58	0.57	0.57	0.57	0.39	0.46
Models with minimum RMSFEs	AR Model	AR Model	AR Model	Litterman Model 2	Litterman Model 2	Sims-Zha Normal Wishart Optimal Model	Sims-Zha Normal Wishart Optimal Model	Sims-Zha Normal Wishart Optimal Model	VAR Model Optimal	VAR Model Optimal	VAR Model Optimal	VAR Model Optimal	Litterman Model 4	Litterman Model 4	VAR Model 3	Litterman Model 4	Litterman Model 4	Litterman Model 4	VAR Model 4	VAR Model 4	VAR Model 4	VAR Model 4	VAR Model 4	VAR Model 4		

Source: Constructed by author
Models shaded with deep green color highlight lower forecast error, while those in a lighter tone indicate higher forecast error.

Appendix B2: RMSFEs of by category-based models

Models	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M	11M	12M	13M	14M	15M	16M	17M	18M	19M	20M	21M	22M	23M	24M	Average of 12 months	Average of 24 months
AR Model	0.35	0.43	0.41	0.43	0.46	0.47	0.48	0.50	0.48	0.49	0.51	0.52	0.53	0.52	0.53	0.66	0.66	0.66	0.66	0.66	0.65	0.64	0.64	0.65	0.46	0.54
VAR Model 1	0.48	0.56	0.53	0.42	0.41	0.44	0.48	0.53	0.54	0.55	0.55	0.57	0.59	0.58	0.57	0.66	0.67	0.67	0.67	0.66	0.66	0.66	0.65	0.65	0.51	0.57
VAR Model 2	0.73	0.61	0.60	0.41	0.41	0.45	0.43	0.43	0.41	0.43	0.45	0.48	0.48	0.43	0.44	0.59	0.62	0.62	0.61	0.61	0.61	0.60	0.60	0.60	0.49	0.53
VAR Model 3	0.52	0.65	0.61	0.49	0.48	0.46	0.50	0.54	0.55	0.55	0.54	0.57	0.58	0.58	0.58	0.66	0.67	0.67	0.67	0.66	0.66	0.66	0.65	0.65	0.54	0.59
VAR Model 4	0.41	0.53	0.53	0.44	0.43	0.48	0.51	0.54	0.53	0.53	0.54	0.57	0.58	0.58	0.56	0.66	0.67	0.67	0.67	0.66	0.66	0.65	0.65	0.65	0.50	0.57
VAR Model 5	0.43	0.57	0.60	0.45	0.44	0.49	0.52	0.55	0.56	0.57	0.58	0.60	0.61	0.61	0.59	0.67	0.69	0.69	0.69	0.68	0.68	0.67	0.67	0.67	0.53	0.59
VAR Model Optimal	0.61	0.61	0.58	0.43	0.40	0.39	0.36	0.37	0.37	0.38	0.40	0.43	0.44	0.43	0.44	0.59	0.62	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.44	0.50
Litterman Model 1	0.47	0.56	0.53	0.44	0.42	0.44	0.48	0.52	0.53	0.54	0.54	0.57	0.58	0.58	0.55	0.64	0.67	0.66	0.67	0.66	0.65	0.63	0.65	0.65	0.50	0.57
Litterman Model 2	0.68	0.59	0.55	0.40	0.39	0.41	0.41	0.40	0.41	0.43	0.42	0.46	0.47	0.46	0.47	0.59	0.63	0.62	0.63	0.61	0.61	0.62	0.61	0.60	0.46	0.52
Litterman Model 3	0.50	0.62	0.57	0.47	0.45	0.44	0.45	0.51	0.53	0.52	0.52	0.54	0.56	0.55	0.57	0.64	0.65	0.69	0.66	0.66	0.67	0.67	0.65	0.65	0.51	0.57
Litterman Model 4	0.40	0.50	0.49	0.41	0.41	0.45	0.48	0.51	0.51	0.51	0.52	0.54	0.55	0.55	0.55	0.65	0.67	0.67	0.66	0.66	0.66	0.66	0.64	0.65	0.48	0.55
Litterman Model 5	0.42	0.54	0.56	0.43	0.42	0.46	0.48	0.52	0.53	0.54	0.55	0.57	0.59	0.57	0.57	0.66	0.68	0.69	0.68	0.68	0.67	0.68	0.66	0.67	0.50	0.58
Litterman Optimal Model	0.58	0.59	0.55	0.41	0.39	0.38	0.37	0.37	0.38	0.39	0.40	0.44	0.46	0.46	0.46	0.62	0.64	0.63	0.62	0.62	0.64	0.63	0.62	0.63	0.44	0.51
Sims-Zha Model 1	0.47	0.56	0.53	0.43	0.42	0.44	0.48	0.53	0.54	0.55	0.55	0.57	0.58	0.58	0.57	0.65	0.66	0.65	0.66	0.66	0.65	0.64	0.64	0.65	0.50	0.57
Sims-Zha Model 2	0.69	0.60	0.56	0.40	0.40	0.41	0.40	0.41	0.41	0.43	0.44	0.46	0.47	0.44	0.45	0.58	0.62	0.61	0.60	0.61	0.60	0.60	0.60	0.60	0.47	0.52
Sims-Zha Model 3	0.53	0.65	0.60	0.48	0.44	0.43	0.47	0.52	0.53	0.53	0.53	0.54	0.55	0.55	0.56	0.64	0.66	0.65	0.66	0.65	0.65	0.64	0.61	0.63	0.52	0.57
Sims-Zha Model 4	0.42	0.59	0.56	0.47	0.45	0.50	0.55	0.58	0.55	0.55	0.55	0.56	0.57	0.57	0.57	0.64	0.67	0.66	0.67	0.66	0.65	0.65	0.63	0.64	0.53	0.58
Sims-Zha Model 5	0.46	0.57	0.57	0.45	0.41	0.45	0.48	0.52	0.53	0.55	0.55	0.56	0.57	0.57	0.57	0.65	0.68	0.67	0.67	0.67	0.66	0.66	0.64	0.66	0.51	0.57
Sims-Zha Normal Wishart Optimal Model	0.58	0.60	0.55	0.41	0.40	0.37	0.35	0.36	0.37	0.38	0.40	0.43	0.44	0.44	0.47	0.61	0.65	0.62	0.62	0.62	0.61	0.62	0.61	0.62	0.43	0.51
Minimum RMSFEs of models	0.35	0.43	0.41	0.40	0.39	0.37	0.35	0.36	0.37	0.38	0.40	0.43	0.44	0.43	0.44	0.58	0.62	0.61	0.60	0.61	0.60	0.60	0.60	0.60	0.39	0.47
Models with minimum RMSFEs	AR Model	AR Model	AR Model	Litterman Model 2	Litterman Model 2	Sims-Zha Optimal Model	Sims-Zha Optimal Model	Sims-Zha Optimal Model	VAR Model Optimal	VAR Model Optimal	VAR Model Optimal	VAR Model Optimal	VAR Model Optimal	VAR Model Optimal	VAR Model 2	Sims-Zha Model 2	VAR Model Optimal	Sims-Zha Model 2	Sims-Zha Model 2	VAR Model Optimal	Sims-Zha Model 2	Sims-Zha Model 2	Sims-Zha Model 2	Litterman Model 2		

Source: Constructed by author
Models shaded with deep green color highlight lower forecast error, while those in a lighter tone indicate higher forecast error.

Appendix C1: Combination forecast of incremental models for periods

Models	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M	11M	12M	13M	14M	15M	16M	17M	18M	19M	20M	21M	22M	23M	24M
AR Model	0.09	0.07	0.08	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 1	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 2	0.04	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 3	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 4	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
VAR Model 5	0.04	0.04	0.05	0.04	0.04	0.04	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model Optimal	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Model 1	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Model 2	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Model 3	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.06	0.05	0.06	0.05	0.05	0.05
Litterman Model 4	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.06	0.06
Litterman Model 5	0.05	0.05	0.05	0.05	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Optimal Model	0.06	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 1	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 2	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 3	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.05	0.06	0.06	0.06	0.06	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 4	0.05	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.05	0.06	0.06	0.05	0.06	0.05	0.06	0.06	0.06	0.06
Sims-Zha Model 5	0.04	0.05	0.05	0.05	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Normal Wishart Optimal Model	0.06	0.05	0.06	0.06	0.06	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

Source: Constructed by author

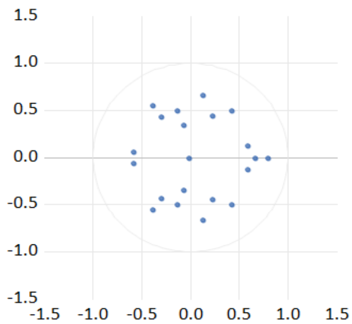
Appendix C2: Combination forecast of category-based models for periods

Models	1M	2M	3M	4M	5M	6M	7M	8M	9M	10M	11M	12M	13M	14M	15M	16M	17M	18M	19M	20M	21M	22M	23M	24M
AR Model	0.07	0.07	0.07	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 2	0.04	0.05	0.05	0.06	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
VAR Model 3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 4	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model 5	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
VAR Model Optimal	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.06
Litterman Model 1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Model 2	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.06	0.06	0.05	0.05	0.06
Litterman Model 3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Model 4	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Model 5	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Litterman Optimal Model	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.05
Sims-Zha Model 1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 2	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Sims-Zha Model 3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 4	0.06	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Model 5	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Sims-Zha Normal Wishart Optimal Model	0.04	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.06	0.06	0.05	0.06	0.05	0.05	0.05

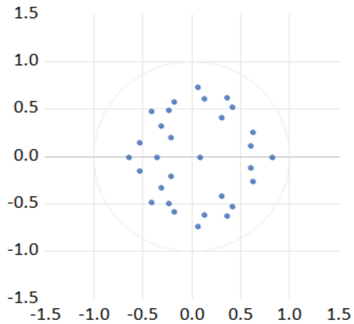
Source: Constructed by author

Appendix D: Inverse Roots of AR Characteristic Polynomial for incremental models

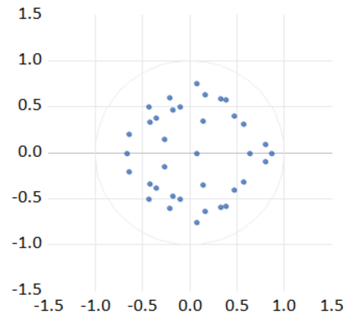
Litterman/Minnesota Model 1



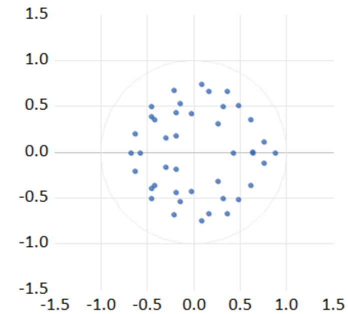
Litterman/Minnesota Model 2



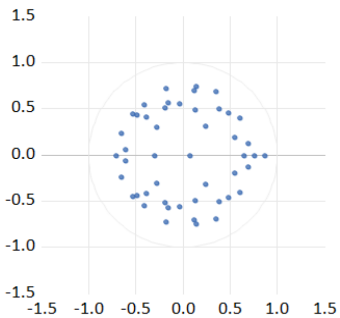
Litterman/Minnesota Model 3



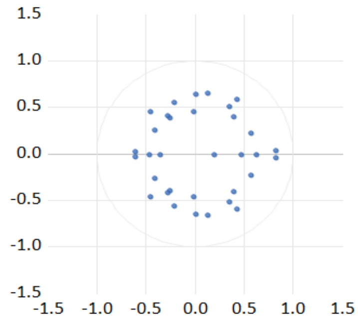
Litterman/Minnesota Model 4



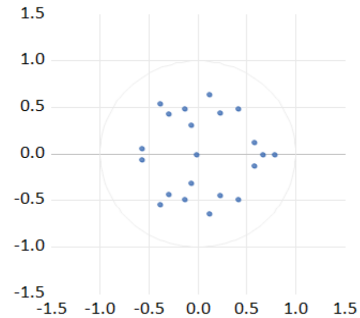
Litterman/Minnesota Model 5



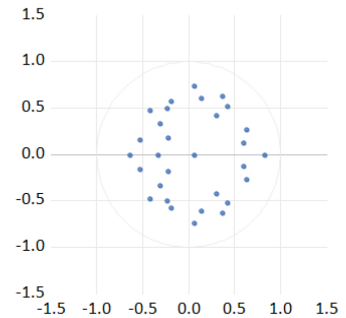
Litterman/Minnesota Optimal Model



Sim-Zha (NW) Model 1



Sim-Zha (NW) Model 2



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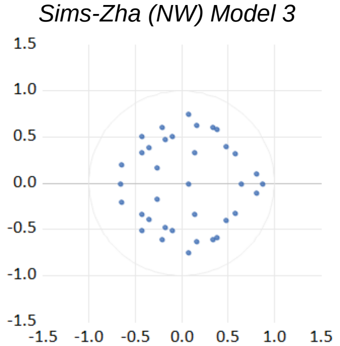

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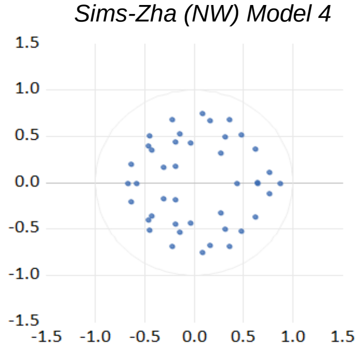

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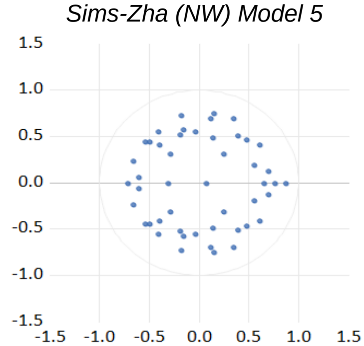
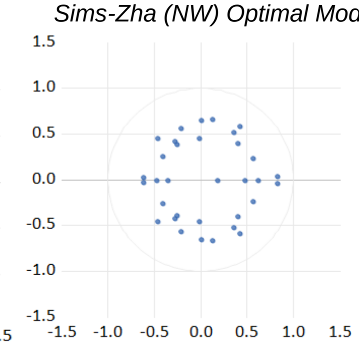
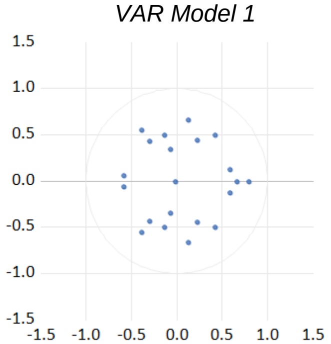


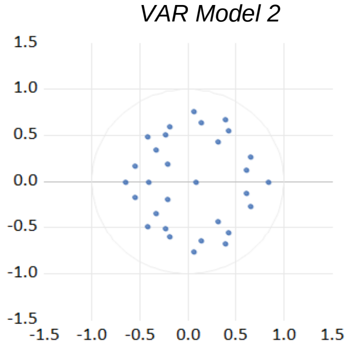
Sims-Zha (NW) Optimal Model



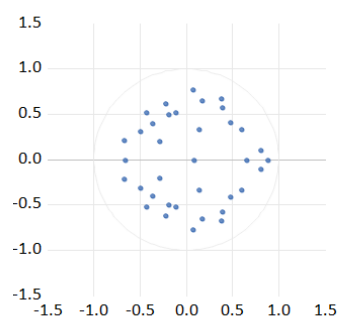
VAR Model 1



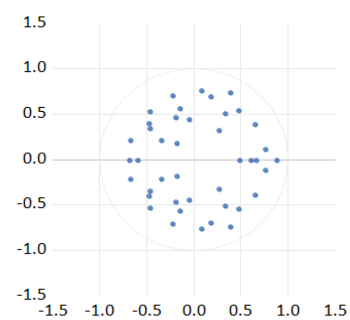
VAR Model 2



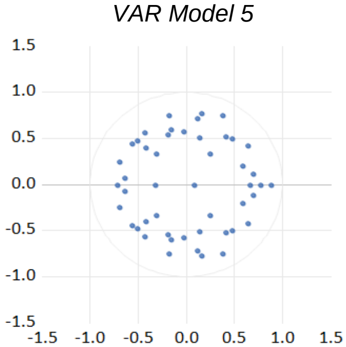
VAR Model 3



VAR Model 4



VAR Model 5



VAR Model 5